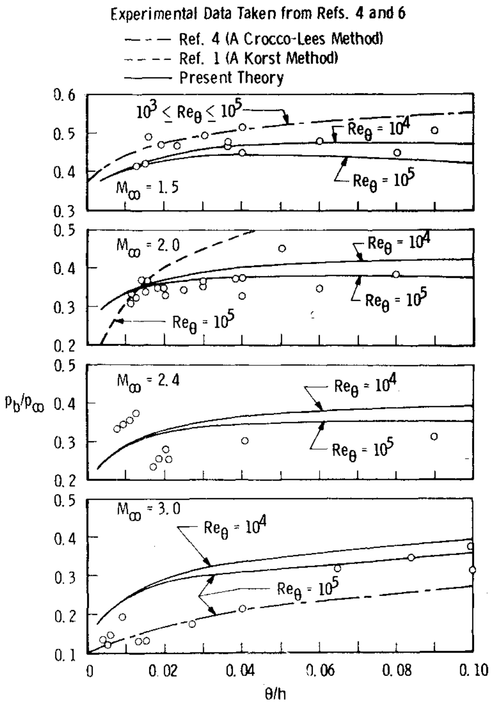

Fig. 3 Base pressure vs initial momentum thickness.

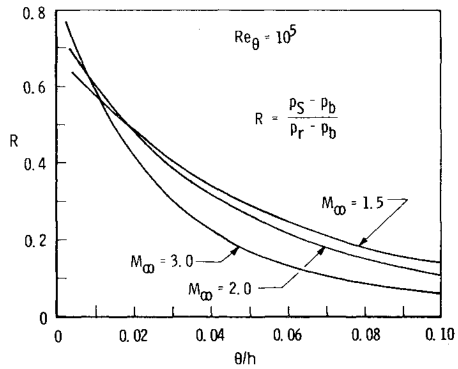

Fig. 4 Recompression factor vs momentum thickness.

of the calculations are compared with experiment in Fig. 3. The power-law exponent was taken from Ref. 4 where a correlation between Re_θ and $1/n$ is presented. The data are taken from that gathered by McDonald[6] and from Ref. 4.

Nash[7] defines a recompression factor as:

$$R = (p_S - p_b)/(p_r - p_b) \tag{10}$$

This factor, corresponding to the theoretical results of Fig. 3, is presented in Fig. 4.

Acknowledgment

The research reported herein was performed by the Arnold Engineering Development Center, Air Force Systems Command. Work and analysis for this research was done by personnel of ARO, Inc., a Sverdrup Corporation Company, operating contractor of AEDC.

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Singularity at the Trailing Edge of a Swept Wing

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Introduction

IN calculations of three-dimensional transonic flows, the most commonly applied method has been that of transonic small-perturbation theory, solving the resulting equations by finite differences. Examples of the solution of the full equations of motion applying the boundary conditions exactly, have been few, e.g., Duck,[1] Jameson,[2] Clapworthy,[3] Forsey and Carr.[4] The results of some of these are applicable only in certain specialized cases.

The general philosophy of these methods is to introduce a coordinate system defined by the shape of the wing, so that the boundary conditions may be satisfied exactly (in the numerical sense). One manner of doing this, which has been used successfully in Refs. 3 and 4, is to define the set of vertical planes that are parallel to the flow, to be coordinate surfaces; the transformation to body-defined coordinates is then completed by performing the conformal mappings of the wing section on to a unit circle in each of these planes. This coordinate system will, in general, be nonorthogonal.

The existence of numerical conformal mapping techniques means that the mappings will usually be performed numerically. However, when the method was used for swept wings with a Kármán-Trefftz profile, the mappings for which can be expressed analytically, it was discovered that there are certain difficulties associated with the trailing edge. The use of numerical conformal mappings would, almost certainly, result in these difficulties being overlooked, and it is the purpose of this Note to draw attention to their existence.

Coordinate System

We consider a symmetrical wing with a Kármán-Trefftz profile, swept at an angle Λ, at zero incidence to the freestream. The Cartesian coordinate system x^i has x^1 spanwise, x^2 upstream, and x^3 vertically upward. From this, we transform to a nonorthogonal reference system ξ^i in which the center section is represented by $\xi^1 = 0$, planes parallel to it by $\xi^1 =$ constant, and the wing surface by $\xi^2 = 0$.

Received Sept. 20, 1978.
Index categories: Subsonic Flow; Transonic Flow.
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For an untapered wing with a straight leading edge, the transformation has the general form

$$x^1 = \xi^1, \quad x^2 + ix^3 = -\mu\xi^1 + f(\zeta) \tag{1}$$

where $\mu = \tan\Lambda$ and $\zeta = \xi^2 + i\xi^3$. The form of f depends upon the shape of the profile (see Ref. 5). The limits of the coordinates are

$$0 \le \xi^1 \le \infty, \quad 0 \le \xi^2 \le \infty, \quad 0 \le \xi^3 \le 2\pi$$

applying symmetry in the spanwise (x^1) direction.

Flow Equations

The precise forms of the equations of motion and the boundary conditions in the nonorthogonal system are not important for the present discussion (they are set out in Ref. 3). The solution is obtained using finite differences, employing line relaxation either along "spokes" (i.e., lines ξ^1, ξ^3 constant) or along "rings" (i.e., lines ξ^1, ξ^2 constant). Because the trailing edge is a singular line for the transformation just described, the details differ according to the method used—the difficulty to be outlined occurs in both, but is most easily demonstrated for relaxation along spokes.

We consider only incompressible flow; compressibility can be shown not to affect the main features of the flow at the trailing edge. Along the spoke through the trailing edge ($\xi^3 = \pi$) the equation of incompressible flow simplifies to

$$A^2\Phi_{11} + (1+\mu^2)\Phi_{22} + \Phi_{33} = 2\mu A\Phi_{12} + \frac{\mu^2 R}{A^4} V_2 \tag{2}$$

where

$$\Phi_{ij} = \frac{\partial^2\Phi}{\partial\xi^i \partial\xi^j}, \quad V_2 = \frac{\partial\Phi}{\partial\xi^2}, \quad A = \left|\frac{df}{d\zeta}\right|$$

Φ is the total velocity potential and R (a function of ξ^2, ξ^3) is defined in Refs. 3 and 5.

It is found most convenient to work henceforth in the χ plane where χ is an intermediate complex coordinate of the transformation described in Eq. (1) and defined by

$$\chi = e^{\zeta} + \epsilon, \quad 0 < \epsilon < 1$$

so that in the χ plane, the wing profile ($\xi^2 = 0$) becomes a unit circle, center (ϵ,0) (see Fig. 1).

Near the trailing edge T, $l_1 \to 0$. In this region let $l_1 = \delta$, then it can be shown that along $\xi^3 = \pi$, $\delta = \xi^2$, and that Eq. (2) simplifies to

$$(1+\mu^2)\Phi_{22} + \Phi_{33} = \frac{\mu^2 k(k-1)}{(2l)^{k-1}} \left(\frac{V_2}{A}\right) \frac{1}{\delta^{2-k}} \tag{3}$$

where $l = 1 - \epsilon$ and k ($0 \le k \le 2$) is a parameter of the Kármán-Trefftz transformation defined by the trailing-edge angle Ω of the profile

$$\Omega = (2-k)\pi$$

The boundary condition on $\xi^2 = 0$ becomes, at the trailing edge,

$$V_2/A = \mu V_1/1(+\mu^2) \tag{4}$$

Now V_2/A and $V_1/\sqrt{1+\mu^2}$ are the components of the fluid velocity along the lines ξ^1, ξ^3 constant and ξ^2, ξ^3 constant, respectively. Hence, it can be expected that these remain finite and nonzero at the trailing edge. Therefore, for $k < 2$, the right-hand side of Eq. (3) diverges at the trailing edge ($\delta \to 0$) and the finite-difference scheme breaks down.

For an unswept wing $\mu = 0$, and the singular expression is not present; similarly for a cusped trailing edge ($k = 2$) there is no singular term. Moreover, at the centerline, $V_1 = 0$, so that the application of the boundary condition in Eq. (4) means that the right-hand side of Eq. (3) disappears.

Nevertheless, in general, the singular term will be present. For a wing with a trailing-edge angle of 9 deg, $k = 1.95$, and the singular term in Eq. (3) is of order $\delta^{-0.05}$. In the region of the trailing edge, the derivative of the mapping function $df/d\zeta$ will be of order $\delta^{0.95}$; that is, it is very nearly linear, and $d^2f/d\zeta^2$ will be of order $\delta^{-0.05}$. It is the behavior of the latter term which is the cause of the singularity. The use of numerical conformal mappings would result, unless great care were taken, in $df/d\zeta$ being treated as linear and some finite value being assigned to $d^2f/d\zeta^2$.

Local Solution

The appearance of the singular term in Eq. (3) is due to the choice of a coordinate system in which the wing is a coordinate surface. It is present when the same coordinates are used for the calculation of incompressible flow past an infinite swept wing, for which solutions are readily obtained by applying a conformal mapping in sections normal to the leading edge. We use this fact to produce a local solution for the velocity potential in the trailing-edge region.

Introduce a Cartesian coordinate system, y_i centered at some point P on the trailing edge with y_1 along the trailing edge, y_2 normal to the trailing edge, and y_3 vertically upwards. The wing is represented by a wedge of angle $\omega = \frac{1}{2}\Omega$ mounted on a wall $y_3 = 0$ representing the plane of symmetry (see Fig. 2).

If c is a real constant, the transformation

$$\sigma^k = cZ \tag{5}$$

where $Z = y_2 + iy_3$ maps this region into a right-angled corner in the σ plane.

For flow in such a corner, the complex potential is

$$W - W_P = (\Phi - \Phi_P) + i\Psi = B\sigma^2 \tag{6}$$

where B is a real constant and W_P, Φ_P are the values at P. Thus,

$$\Phi - \Phi_P = BRe[(cZ)^{2/k}] \tag{7}$$

The foregoing is, strictly speaking, relevant only to incompressible flows. However, as stated earlier, compressibility effects in the trailing-edge region are negligible and the result can be applied more generally.

The local solution is fitted into the finite-difference scheme by applying Eq. (6) at the grid point nearest the trailing edge along the line of symmetry ($\psi = 0$) and at the nearest point

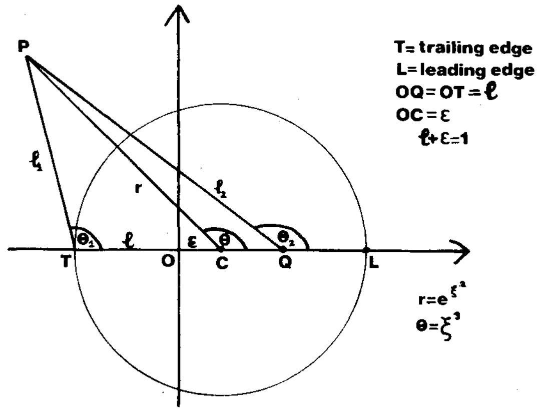

Fig. 1 Geometry of χ plane.

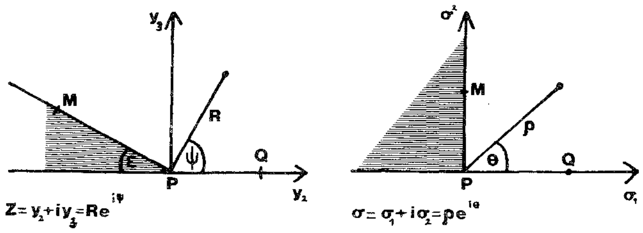

Fig. 2 Mapping of the trailing-edge region.

along the wing ($\psi=\pi-\omega$). From these two equations, we eliminate the constant $Bc^{2/k}$ to calculate Φ_P.

Further terms could be incorporated into Eqs. (5) and (6) to obtain a higher-order solution, in which case Eq. (7) would have to be applied at additional grid points to eliminate the extra constants introduced.

Concluding Remarks

The use of a coordinate system, which has already been successfully used in the computation of transonic flows past swept wings, has been shown to give rise to a singular term in the equation of motion at a sharp swept trailing edge. An alternative to the present approach using a local solution appears to be to calculate the potential at the trailing edge by extrapolation along the wing surface (see Forsey and Carr[4]). However, the linear extrapolation used assumes that the potential has a linear behavior in ξ^3, a quality which it can be shown not to possess. In addition, the equation of motion is not satisfied at the trailing edge in that case.

How sensitive the calculated flow is to differences in the numerical treatment at the trailing edge is open to conjecture, but it seems worthwhile to draw the attention of other investigators to a possible source of error, and to suggest a way of avoiding it.

Acknowledgment

Thanks are due to the Science Research Council for financial support during the period that the author was at the University of Southampton, and to K. W. Mangler and J. H. B. Smith for their helpful comments.

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Modes of Turbulent Vortex Shedding from a Pipe-Jet System in a Cross-Flow

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I. Introduction

A PAPER by Moussa, et al[1] showed the characteristics of vortex shedding from a pipe-jet system in a cross-flow. The emphasis was on shedding from the jet. A few downstream measurements made at vertical positions above and below the plane of the pipe orifice led to the conclusion that the pipe and jet constituted a single shedding system. In other experiments, this system was broken into two isolated parts by means of a flat plate placed in the plane of the orifice, and vortex shedding occurred independently from the jet and from the pipe.

This Note presents the results of more extensive measurements of the unified pipe-jet system in which no plate was used in the plane of the pipe orifice. It has been found that the pipe and jet shed vortices at significantly different rates. The differences depend on the ratio of jet speed U_j to the speed of the undisturbed cross-flow U_∞. The existence of two different shedding systems for the jet and for the pipe has led to naming them the "jet mode" and the "pipe mode," respectively.

When observations were made by varying only the z coordinate parallel to the pipe axis, it was found that in the jet region far above the pipe orifice there was only one shedding frequency, that of the jet mode. Similarly, for positions well below the pipe orifice only the frequency of the pipe mode was found. However, for several pipe diameters above and below the orifice, two shedding frequencies were observed; Fig. 1 is an example. One of these frequencies is identical with that of the pipe mode; the other identical with that of the jet mode. In the range in which frequencies are observed, the modes are said to "overlap."

Measurements made over the full range of the z coordinate from pipe base to jet locations at which shedding disappeared, and over a wide range of U_j/U_∞, showed that four different modes could be defined. Each mode overlapped the z-adjacent mode. The jet and pipe modes occur distinguishably for $U_j/U_\infty>5$. For $U_j/U_\infty<1$, a weak jet condition, there is no discernible jet mode, but because of the importance of the end effect, the end of the pipe from the lip to a distance several diameters below the lip sheds vortices at a slower rate than the main body of the pipe. This new mode is called the "end mode." The fourth mode, called the "base mode," overlaps the pipe mode and occurs in the region where the pipe meets the floor of the wind tunnel. The shedding frequency of the base mode is lower than that of the pipe mode.

II. Apparatus and Measurements

The apparatus has been described in detail in Ref. 1. The wind tunnel has a cross section 1.22 m high by 0.61 m wide. A perpendicular pipe, $D=2.54$ cm o.d. and 2.36 cm i.d., comes through the tunnel floor to a height of 46 cm. A turbulent jet issues from the open end of the pipe. A speed-sensing hot wire probe is connected to an anemometer circuit whose output passes through a linearizer to a narrow band, real-time spectrum analyzer. This, in turn, drives an x-y recorder after an averaging time of 5 min has elapsed. The incremental fre-

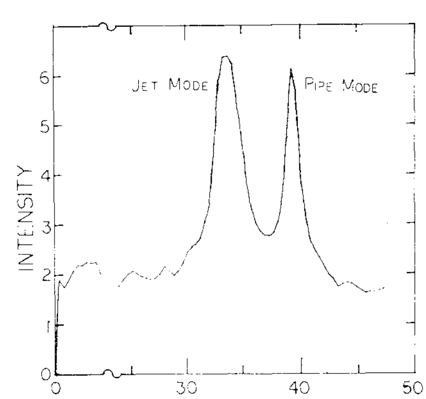

Fig. 1 Typical trace of power spectrum showing jet mode, pipe mode, and a reference turbulence level near zero frequency. The intensity scale is abitrary.

Received Oct. 2, 1978.
Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.
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